Further Centres of Mass

Questions

Q1.

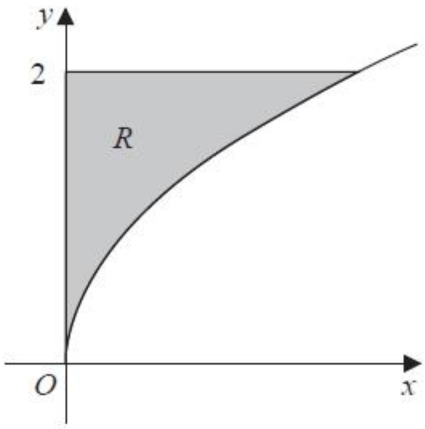


Figure 4

The region R, shown shaded in Figure 4, is bounded by part of the curve with equation $y^2 = 2x$,
the line with equation $y = 2$ and the y-axis. The unit of length on both axes is one centimetre.
A uniform solid, S, is formed by rotating R through 360° about the y-axis.

Given that the volume of S is $\frac{8}{5}\pi$ cm^3,

(a) show that the centre of mass of S is $\frac{1}{3}$ cm from its plane face.

(4)

A uniform solid cylinder, C, has base radius 2 cm and height 4 cm. The cylinder C is attached to S so that the plane face of S coincides with a plane face of C, to form the paperweight P, shown in Figure 5. The density of the material used to make S is three times the density of the material used to make C.

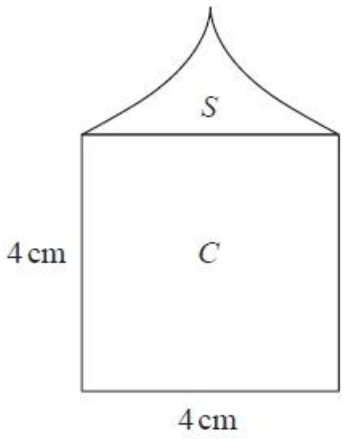


Figure 5

The plane face of P rests in equilibrium on a desk lid that is inclined at an angle $\theta°$ to the horizontal. The lid is sufficiently rough to prevent P from slipping. Given that P is on the point of toppling,

(b) find the value of θ.

(7)

(Total for question = 11 marks)

Q2.

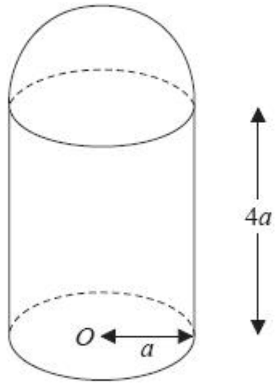


Figure 3

A thin uniform right hollow cylinder, of radius a and height $4a$, has a base but no top. A thin uniform hemispherical shell, also of radius a, is made of the same material as the cylinder. The hemispherical shell is attached to the open end of the cylinder forming a container C. The open circular rim of the cylinder coincides with the rim of the hemispherical shell. The centre of the base of C is O, as shown in Figure 3.

(a) Find the distance from O to the centre of mass of C.

(6)

The container is placed with its circular base on a plane which is inclined at $\theta°$ to the horizontal. The plane is sufficiently rough to prevent C from sliding. The container is on the point of toppling.

(b) Find the value of θ.

(3)

(Total for question = 9 marks)

Q3.

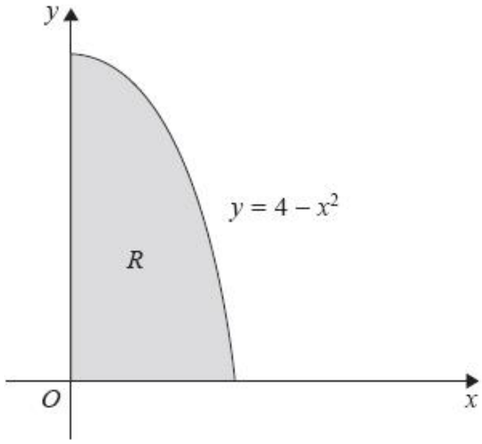


Figure 1

A uniform lamina is in the shape of the region *R*. Region *R* is bounded by the curve with equation $y = 4 - x^2$, the positive *x*-axis and the positive *y*-axis, as shown shaded in Figure 1.

Use algebraic integration to find the *x* coordinate of the centre of mass of the lamina.

(7)

(Total for question = 7 marks)

Q4.

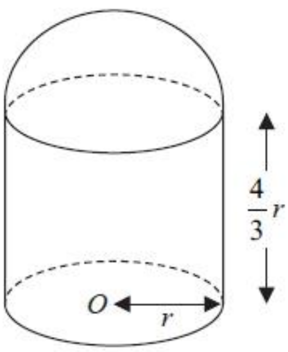


Figure 3

A uniform solid cylinder of base radius *r* and height $\frac{4}{3}r$ has the same density as a uniform solid hemisphere of radius *r*. The plane face of the hemisphere is joined to a plane face of the cylinder to form the composite solid *S* shown in Figure 3.
The point *O* is the centre of the plane face of *S*.

(a) Show that the distance from *O* to the centre of mass of *S* is $\frac{73}{72}r$

(4)

The solid *S* is placed with its plane face on a rough horizontal plane. The coefficient of friction between *S* and the plane is μ. A horizontal force *P* is applied to the highest point of *S*. The magnitude of *P* is gradually increased.

(b) Find the range of values of μ for which *S* will slide before it starts to tilt.

(5)

(Total for question = 9 marks)

Q5.

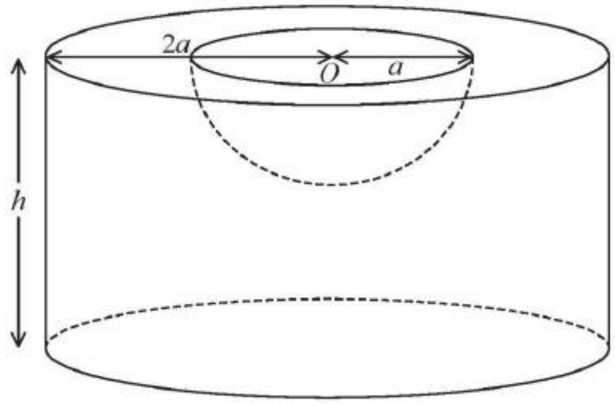


Figure 2

A uniform solid cylinder has radius 2*a* and height *h* ($h > a$).

A solid hemisphere of radius *a* is removed from the cylinder to form the vessel *V*.

The plane face of the hemisphere coincides with the upper plane face of the cylinder.

The centre *O* of the hemisphere is also the centre of the upper plane face of the cylinder, as shown in Figure 2.

(a) Show that the centre of mass of *V* is $\dfrac{3(8h^2 - a^2)}{8(6h - a)}$ from *O*.

(5)

The vessel *V* is placed on a rough plane which is inclined at an angle Φ to the horizontal.

The lower plane circular face of *V* is in contact with the inclined plane.

Given that $h = 5a$, the plane is sufficiently rough to prevent *V* from slipping and *V* is on the point of toppling,

(b) find, to three significant figures, the size of the angle Φ.

(4)

(Total for question = 9 marks)

Q6.

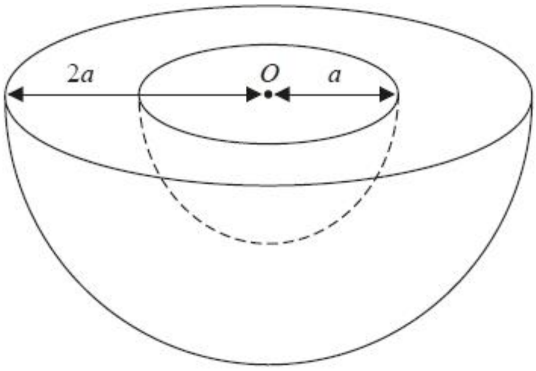


Figure 2

A uniform solid hemisphere H has radius $2a$. A solid hemisphere of radius a is removed from the hemisphere H to form a bowl. The plane faces of the hemispheres coincide and the centres of the two hemispheres coincide at the point O, as shown in Figure 2.

The centre of mass of the bowl is at the point G.

(a) Show that $OG = \dfrac{45a}{56}$

(4)

Figure 3 below shows a cross-section of the bowl which is resting in equilibrium with a point P on its curved surface in contact with a rough plane. The plane is inclined to the horizontal at an angle α and is sufficiently rough to prevent the bowl from slipping. The line OG is horizontal and the points O, G and P lie in a vertical plane which passes through a line of greatest slope of the inclined plane.

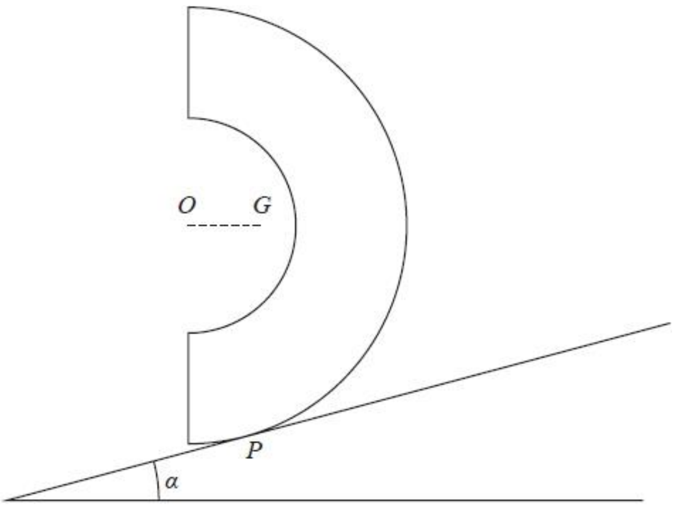


Figure 3

(b) Find the size of α, giving your answer in degrees to 3 significant figures.

(2)

(Total for question = 6 marks)

Q7.

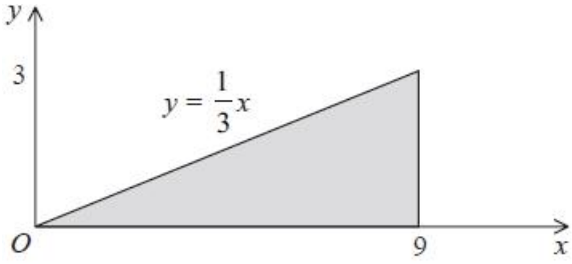


Figure 4

The shaded region shown in Figure 4 is bounded by the *x*-axis, the line with equation $x = 9$ and the line with equation $y = \frac{1}{3}x$. This shaded region is rotated through 360° about the *x*-axis to form a solid of revolution. This solid of revolution is used to model a solid right circular cone of height 9 cm and base radius 3 cm.

The cone is non-uniform and the mass per unit volume of the cone at the point (x, y, z) is λx kg cm^{-3}, where $0 \leq x \leq 9$ and λ is constant.

(a) Find the distance of the centre of mass of the cone from its vertex.

(6)

A toy is made by joining the circular plane face of the cone to the circular plane face of a uniform solid hemisphere of radius 3 cm, so that the centres of the two plane surfaces coincide.

The weight of the cone is *W* newtons and the weight of the hemisphere is *kW* newtons.

When the toy is placed on a smooth horizontal plane with any point of the curved surface of the hemisphere in contact with the plane, the toy will remain at rest.

(b) Find the value of *k*

(4)

(Total for question = 10 marks)

Q8.

A flagpole, AB, is 4 m long. The flagpole is modeled as a non-uniform rod so that, at a distance x metres from A, the mass per unit length of the flagpole, m kg m^{-1}, is given by $m = 18 - 3x$.

(a) Show that the mass of the flagpole is 48 kg.

(3)

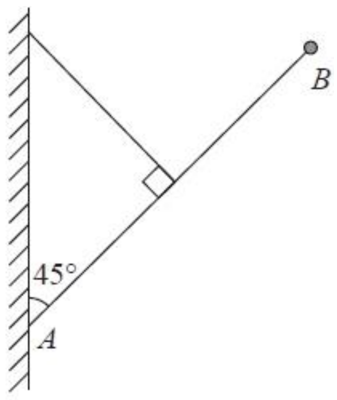


Figure 3

The end A of the flagpole is fixed to a point on a vertical wall. A cable has one end attached to
the midpoint of the flagpole and the other end attached to a point on the wall that is vertically above A. The cable is perpendicular to the flagpole. The flagpole and the cable lie in the same
vertical plane that is perpendicular to the wall. A small ball of mass 4 kg is attached to the flagpole at B. The cable holds the flagpole and ball in equilibrium, with the flagpole at 45° to the wall, as shown in Figure 3.

The tension in the cable is T newtons.

The cable is modelled as a light inextensible string and the ball is modeled as a particle.

(b) Using the model, find the value of T.

(8)

(c) Give a reason why the answer to part (b) is not likely to be the true value of T.

(1)

(Total for question = 12 marks)

Q9.

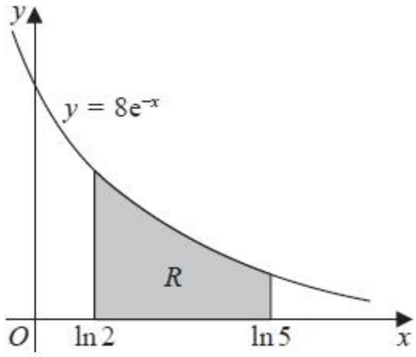


Figure 1

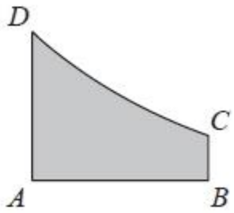


Figure 2

A uniform plane figure R, shown shaded in Figure 1, is bounded by the x-axis, the line with equation $x = \ln 5$, the curve with equation $y = 8e^{-x}$ and the line with equation $x = \ln 2$. The unit of length on each axis is one metre.

The area of R is 2.4 m^2

The centre of mass of R is at the point with coordinates $(\bar{x}, \bar{y})$.

(a) Use algebraic integration to show that $\bar{y}$ = 1.4

(4)

Figure 2 shows a uniform lamina $ABCD$, which is the same size and shape as R. The lamina is freely suspended from C and hangs in equilibrium with CB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ

(6)

(Total for question = 10 marks)

Q10.

Numerical (calculator) integration is not acceptable in this question.

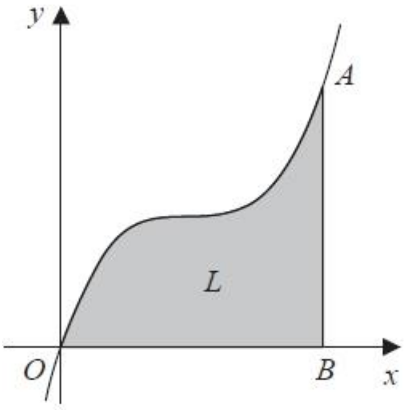


Figure 2

The shaded region *OAB* in Figure 2 is bounded by the *x*-axis, the line with equation

$x = 4$ and the curve with equation $y = \frac{1}{4}(x-2)^3 + 2$. The point *A* has coordinates (4, 4)

and the point *B* has coordinates (4, 0).

A uniform lamina *L* has the shape of *OAB*. The unit of length on both axes is one centimetre. The centre of mass of *L* is at the point with coordinates $(\bar{x}, \bar{y})$.

Given that the area of *L* is 8 cm^2,

(a) show that $\bar{y} = \frac{8}{7}$

(4)

The lamina is freely suspended from *A* and hangs in equilibrium with *AB* at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.

(7)

(Total for question = 11 marks)

Mark Scheme – Further Centres of Mass

Q1.

Question	Scheme	Marks	AOs
(a)	$\int \pi x^2 y \mathrm{d}y = \pi \int \frac{y^5}{4} \mathrm{d}y = \pi \left[\frac{y^6}{24} \right]_0^2$	M1	3.4
	$= \frac{64\pi}{24} \left(= \frac{8\pi}{3} \right)$	A1	1.1b
	$\Rightarrow \bar{y} = \left(\text{their } \frac{8\pi}{3} \right) \times \frac{5}{8\pi} \left(= \frac{5}{3} \right)$	M1	3.1a
	Distance from plane face $= 2 - \frac{5}{3} = \frac{1}{3}$ *	A1*	2.2a
		(4)	
(b)	<table><tr><td></td><td>Mass ratio</td><td>c of m from base</td></tr><tr><td>S</td><td>$3 \times \frac{8\pi}{5}$</td><td>$\frac{13}{3}$</td></tr><tr><td>cylinder</td><td>16π</td><td>2</td></tr><tr><td>M</td><td>$(16 + 4.8)\pi$</td><td>d</td></tr></table>	B1	1.2
	Moments about diameter of the base:	M1	2.1
	$\frac{24}{5} \times \frac{13}{3} + 32 = \frac{104}{5} d \left(= \frac{264}{5} \right)$	A1	1.1b
	$d = \frac{264}{104} \left(= \frac{33}{13} \right)$ (cm)	A1	1.1b
	Complete strategy for θ	M1	3.1a
	About to topple: $\tan\theta = \frac{2}{d} \left(= \frac{26}{33} \right)$	A1ft	3.4
	$\theta = 38.2$ (or 38)	A1	2.2a
		(7)	
			(11 marks)

Question	Marks	Marking Guidance
(a)	M1	Use the model to find the moment about the base. Usual rules for integration. Correct limits
	A1	Any equivalent form
	M1	Use the model and volume to find $\bar{y}$: their moments and $\frac{8\pi}{5}$ used correctly
	A1*	Obtain given answer from correct working
	(4)	
(b)	B1	Correct mass ratios seen or implied
	M1	Moments equation. Dimensionally correct. Allow for their mass ratios
	A1	Correct unsimplified equation
	A1	Any equivalent form (2.54 cm)
	M1	Complete strategy to find the position of the centre of mass of the composite body and use trig. to find θ
	A1ft	Trig ratio for a relevant angle. Follow their d
	A1	Correct angle (38 or better) (38.2338....)
	(7)	

Q2.

Question Number	Scheme	Marks
(a)	Ratio of masses: $\pi a^2 \quad 2\pi a \times 4a \quad 2\pi a^2 \quad 11\pi a^2$	M1A1
	Distances: $0 \quad 2a \quad 4\frac{1}{2}a \quad \bar{y}$	B1
	$(0+)2a \times 8 + 4\frac{1}{2}a \times 2 = 11\bar{y}$	M1A1ft
	$\bar{y} = \frac{25}{11}a \ (= 2.272...a)$	A1 (6)
(b)	$\tan\theta° = \frac{a}{\frac{25}{11}a} \left(= \frac{11}{25}\right)$	M1A1ft
	$\theta = 23.749...$ Accept 24 or better	A1 (3) [9]

(a)

M1 Attempt the ratio of the masses of the separate parts. Formulae used must be correct; allow if cylinder has a top as well as a bottom - ignore top - or neither. Similarly if hemisphere has a base. Allow if the base of the cylinder and the curved surface are combined.

A1 Correct ratio seen eg 1:8:2:11 (or any equivalent) (Top of cylinder/base of hemisphere not ignored now; combined curved surface and base for cylinder not allowed.)

B1 Correct distances from O or any other point for the curved surface of the cylinder and the hemispherical shell.

M1 Form a moments equation about their chosen point. Must be dimensionally correct (ie no a^3 seen). Extra terms score M0.

A1ft Correct equation, follow through their mass ratio and distances. 1 or 2 signs may be negative if a point other than O has been used.

A1cao Correct distance from O exact or min 2sf. (Must be obtained from a correct equation.)

ALT Find c of m of cylinder (inc base) first and then combine with hemisphere. All marks available. Award second M when combining with the hemisphere.

NB If c of m of cylinder with base is given as $2a$ then only M1A0B0M0A0A0 available.

(b)

M1 Use $\tan\theta = \frac{a}{\bar{y}}$ or $\frac{\bar{y}}{a}$ with their answer from (a)

A1ft Correct expression for $\tan\theta$ follow through their $\bar{y}$

A1cao Correct value for θ Min 2 sf. (NB Equivalent in radians scores A0)

Q3.

Question Number	Scheme	Marks
	Area $=\int_0^2 (4-x^2)\,\mathrm{d}x=\left[4x-\frac{1}{3}x^3\right]_0^2, \; =\frac{16}{3}$	M1,A1
	$\int xy\,\mathrm{d}x=\int_0^2 (4x-x^3)\,\mathrm{d}x$	M1
	$=\left[2x^2-\frac{1}{4}x^4\right]_0^2, \; =4$	dM1, A1
	$\bar{x}=\frac{\int xy\,\mathrm{d}x}{\int y\,\mathrm{d}x}=4\div\frac{16}{3}, \; =\frac{3}{4}$ oe	M1,A1cso
		[7]

Use of volumes scores 0/7
Ignore any work for $\bar{y}$ whether before or after $\bar{x}$

M1 Using $\int y\,\mathrm{d}x$ with lower limit 0, an attempt at the upper limit and an attempt at algebraic integration.

A1 Correct result for the area. May be implied by a correct final answer. No need to show substitution of either limit providing algebraic integration and attached limits are correct.

M1 Using $\int xy\,\mathrm{d}x$ with the same limits as the first integral

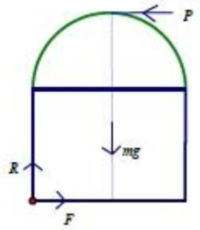
dM1 Attempting the algebraic integration, including substitution of their limits. Depends on the second M mark.

A1 Correct result for this integral. May be implied by a correct final answer.

M1 Using $\bar{x}=\frac{\int xy\,\mathrm{d}x}{\int y\,\mathrm{d}x}$. If a constant for mass/unit area is seen it must be with both integrals or neither.

A1cso Correct final answer.

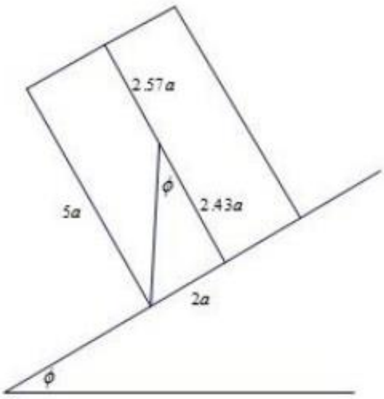
Q4.

Question	Scheme	Marks	AOs
(a)	$\frac{4}{3}\pi r^3 \times \frac{2}{3}r + \frac{2}{3}\pi r^3 \times \left(\frac{4}{3}r + \frac{3}{8}r\right) = \left(\frac{4}{3}+\frac{2}{3}\right)\pi r^3 \times d$	M1	2.1
	$\left(\frac{8}{9}r + \frac{8}{9}r + \frac{1}{4}r = 2d\right)$ $\left(\frac{73}{36}r = 2d\right)$	A1 A1	1.1b 1.1b
	$\Rightarrow d = \frac{73}{72}r$ *	A1*	2.2a
		(4)	
(b)			
	Resolving: $\leftrightarrow F = P$, $\updownarrow R = Mg$, $F_{\max} = \mu R = \mu Mg$	M1	1.1b
	Slides if $P > \mu Mg$	A1	1.2
	Moments: $\frac{7}{3}rP = rMg$ Tilts if $P > \frac{3}{7}Mg$	B1	1.1b
	Comparison of restrictions to determine values of μ	M1	3.1a
	Slides first if $\mu Mg < \frac{3}{7}Mg$, $(0<)\mu < \frac{3}{7}$	A1	2.2a
		(5)	
			(9 marks)

Notes:		
(a)	M1	Moments equation. Dimensionally correct.
	A1 A1	Unsimplified equation with at most one slip Correct unsimplified equation
	A1*	Obtain given result from correct working.
(b)	M1	Resolve and use $F = \mu R$ to find values of P for sliding
	A1	Use the model to form the correct inequality
	B1	Correct inequality for tilting
	M1	Correct comparison of when it tilts and when it slides
	A1	Correct conclusion

Q5.

<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th><th>AOs</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><td></td><td>mass</td><td>c of m from O</td></tr>
<tr><td>cylinder</td><td>$4\pi a^2 h$</td><td>$\frac{h}{2}$</td></tr>
<tr><td>hemisphere</td><td>$\frac{2}{3}\pi a^3$</td><td>$\frac{3}{8}a$</td></tr>
<tr><td>V</td><td>$4\pi a^2 h - \frac{2}{3}\pi a^3$</td><td>$d$</td></tr>
</table>
</td><td></td><td></td></tr>
<tr><td></td><td>Mass ratios</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>Correct distances</td><td>B1</td><td>1.2</td></tr>
<tr><td></td><td>Moments about a diameter through O.</td><td>M1</td><td>2.1</td></tr>
<tr><td></td><td>$4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2\left(2h - \frac{1}{3}a\right) \times d$</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>$d = \dfrac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \dfrac{3(8h^2 - a^2)}{8(6h - a)}$ *</td><td>A1*</td><td>2.2a</td></tr>
<tr><td></td><td></td><td>(5)</td><td></td></tr>
</table>

<table>
<tr><td>(b)</td><td>
</td><td></td><td></td></tr>
<tr><td></td><td>$h = 5a \Rightarrow d = 2.573...a$</td><td>B1</td><td>1.1b</td></tr>
<tr><td></td><td>About to topple so c of m above tipping point</td><td>M1</td><td>2.2a</td></tr>
<tr><td></td><td>$\Rightarrow \tan\phi = \dfrac{2a}{5a - 2.573a}$</td><td>A1ft</td><td>1.1b</td></tr>
<tr><td></td><td>$\phi = 39.5°$ or 0.689 rads.</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td></td><td>(4)</td><td></td></tr>
<tr><td colspan="4">(9 marks)</td></tr>
</table>

Notes:
(a) **B1:** correct mass ratios **B1:** correct distances **M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from O. **A1:** Correct unsimplified equation **A1*:** Deduce the **given answer**. Their working must make it clear how they reached their answer.
(b) **B1:** Distance of com from base **M1:** Condone tan the wrong way up. **A1ft:** Correct unsimplified expression for trig ratio for ϕ following their d **A1:** $39.5°$ or 0.689 rads.

Q6.

Question	Scheme	Marks	AOs
(a)	Mass ratios: $\frac{2}{3}\pi(2a)^3, \frac{2}{3}\pi a^3, \frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3$ (8, 1, 7)	B1	1.2
	Distances: $\frac{3}{8}(2a), \frac{3}{8}a, \bar{x}$	B1	1.2
	$\left(\frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3\right)\bar{x} = \frac{2}{3}\pi(2a)^3 \times \frac{3}{8}(2a) - \frac{2}{3}\pi a^3 \times \frac{3}{8}a$	M1	3.1a
	$\bar{x} = \frac{45a}{56}$ *	A1*	2.2a
		(4)	
(b)	Use of appropriate trig ratio e.g. $\sin\alpha = \frac{\frac{45a}{56}}{2a}$	M1	3.1a
	$\alpha = 23.7°$ (3 sf)	A1	1.1b
		(2)	
		(6 marks)	

Notes:		
a	B1	Correct unsimplified (8, 1, 7)
	B1	Correct unsimplified but distances could be measured from a parallel axis
	M1	All terms needed and must be dimensionally correct
	A1*	Correct answer correctly derived
b	M1	Must be using an appropriate trig ratio
	A1	Cao to 3SF

Q7.

Question	Scheme	Marks	AOs
(a)	Mass of cone $= \int_0^9 \pi y^2 \lambda x \, dx = \pi\lambda \int_0^9 \frac{x^3}{9} dx$	M1	3.4
	$= \pi\lambda \left[\frac{x^4}{36}\right]_0^9 \quad \left(= \frac{729\pi\lambda}{4}(\text{kg})\right)$	A1	1.1b
	Moments: $\int_0^9 \pi y^2 \lambda x \times x \, dx = \pi\lambda \int_0^9 \frac{x^4}{9} dx$	M1	3.4
	$= \frac{\pi\lambda}{45}\left[x^5\right]_0^9 \quad \left(= \frac{\pi\lambda}{5} \times 9^4\right)$	A1	1.1b
	$\Rightarrow d = \dfrac{\frac{\pi\lambda}{5} \times 9^4}{\frac{\pi\lambda}{4} \times 9^3}$	DM1	2.1
	$d = \frac{36}{5} = 7.2(\text{cm})$	A1	1.1b
		(6)	
(b)	Remains at rest $\Rightarrow$ centre of mass at centre of plane surface	B1	2.1
	Moments about diameter of plane surface:	M1	3.1b
	$(9-d)W\left\{=\left(9-\frac{36}{5}\right)W\right\} = \frac{3}{8} \times 3 \times kW$	A1ft	1.1b
	$k = \frac{8}{5}$	A1	1.1b
		(4)	
		(10 marks)	

Notes:	
(a)	NB: Some candidates are confusing the mass and the volume. For the first M1A1: - If they have a correct method for the mass and they tell you that this is mass, award the marks. - If they have a correct method for the mass say nothing, but use it correctly, award the marks. - If they have a correct method for the mass, say nothing, and use it as the moment, then M0 because this implies that they do not think it is the mass.
M1	Use the model to find the mass of the cone. Allow without limits.
A1	Correct integration. Correct limits seen or implied Substitution not required.

	Allow 2/2 if π not seen and consistent with (b) if attempted
M1	Use the model to find the moment of the cone (usual rules for integration) Allow without limits
A1	Correct integration. Correct limits seen or implied Substitution not required. Allow 2/2 if π not seen and consistent with (a)
M1	Complete method to find the distance of the centre of mass from the vertex. A complete method requires the two preceding M marks. They need to get as far as a value for d. If they have a method that comes directly to this stage you might not see the λ or π
A1	Correct only If all you see is $\Rightarrow d = \frac{9^5}{45} \div \frac{9^4}{36}$ or even $\Rightarrow d = \frac{9}{5} \times 4$ then award 6/6 Allow 6/6 if π not seen throughout but otherwise correct
(b)B1	Correct deduction for location of c of m Stated or implied by their moments equation
M1	Moments about diameter of plane face(s) M0 if the moments equation contradicts the centre of mass being on the interface M0 if using volume in place of mass
A1ft	Correct unsimplified equation. Follow their 7.2 Alternative moments equations: Using vertex: $W\bar{x} + kW\left(9 + \frac{3}{8} \times 3\right) = (W + kW) \times 9$ Using base: $W(12 - \bar{x}) + kW\left(3 - 3 \times \frac{3}{8}\right) = (W + kW)3$ If they are working with the axis at an angle they will possibly have trig terms which should cancel.
A1	Correct only

Q8.

Question	Scheme	Marks	AOs
(a)	Total mass $= \int_0^4 (18 - 3x)\,dx$	M1	2.1
	$= \left[18x - \frac{3x^2}{2}\right]_0^4$	A1	1.1b
	$= 18 \times 4 - \frac{3 \times 16}{2}$ $(= 72 - 24)$ $= 48$ (kg) *	A1*	1.1b
		(3)	
(b)	Taking moments about the base: $\int_0^4 x(18 - 3x)\,dx$	M1	3.4
	$= \left[9x^2 - x^3\right]_0^4 (= 80)$	A1	1.1b
	$\Rightarrow 48d = 80$	M1	3.4
	$d = \frac{80}{48} = \frac{5}{3}$ (m)	A1	1.1b
	Complete strategy	M1	3.1b
	$\text{M}(A): 2T = 4\cos 45° \times 4g + \frac{5}{3}\cos 45° \times 48g$	A1ft	1.1b
	$\left(= \frac{96g}{\sqrt{2}}\right)$	A1ft	1.1b
	$T = 333$ or 330	A1	2.2a
		(8)	
(c)	Any appropriate comment e.g. the ball has been modelled as a point mass – its centre could be further from A	B1	3.5b
		(1)	
		(12 marks)	

Question	Marks	Marking Guidance
(a)	M1	Use integration (usual rules) – do not need to see limits at this stage
	A1	(M1 on epen) Correct integration and correct limits seen
	A1*	Show sufficient working to justify given answer
	(3)	

(b)	M1	Use the model to find the moment of the pole and the ball about A (usual rules for integration)
	A1	Correct integration
	M1	Use the model to complete the moments equation. Require their 80 and 48 used correctly
	A1	Any equivalent form
	M1	Complete strategy to find the tension – e.g.locate c of m of the pole and use moments.
	A1ft A1ft	Moments equation with at most one error. Follow their c of m provided not at centre Correct unsimplified moments equation for their c of m not at centre
	A1	Accept $24\sqrt{2}g$, 333 or 330 ISW
	(8)	
(c)	B1	• The mass of the cable has been ignored – unlikely to be negligible if it can hold a pole this long. • The flagpole will be subject to cross winds • The cable might be extensible • The pole might not be rigid NOT: the wall might be rough / smooth Ignore incorrect statements
	(1)	

Q9.

Question	Scheme	Marks	AOs
(a)	$2.4\bar{y} = \frac{1}{2}\int y^2 \mathrm{d}x = \frac{1}{2}\int \{64\mathrm{e}^{-2x}\}\mathrm{d}x$	M1	2.1
	$= -16\left[\mathrm{e}^{-2x}\right]_{\ln 2}^{\ln 5}$	A1	1.1b
	Complete strategy to find $\bar{y}$	M1	3.1a
	$2.4\bar{y} = -16\mathrm{e}^{-\ln 25} + 16\mathrm{e}^{-\ln 4} = \frac{16}{4} - \frac{16}{25} = \frac{84}{25}$, $\bar{y} = \frac{84}{25} \times \frac{10}{24}\left(= \frac{7}{5}\right) = 1.4$ *	A1*	2.2a
		(4)	
(b)	$2.4\bar{x} = \int\left(8x\mathrm{e}^{-x}\right)\mathrm{d}x$	M1	2.1
	$\left(= \left[-8x\mathrm{e}^{-x} - 8\mathrm{e}^{-x}\right]_{\ln 2}^{\ln 5}\right)$		
	$= -\frac{8}{5}(\ln 5 + 1) + \frac{8}{2}(\ln 2 + 1)(= 2.5974.....)$	M1	1.1b
	$\bar{x} = 1.08$	A1	1.1b
	Complete strategy to find θ	M1	3.1a
	$\tan\theta° = \frac{\ln 5 - \bar{x}}{8\mathrm{e}^{-\ln 5} - 1.4}(= 2.63......)$	A1ft	3.4
	$\theta = 69$	A1	1.1b
		(6)	
		(10 marks)	

Notes:		
(a)	M1	Moments equation to obtain terms of the correct form (with or without limits) Allow if area (2.4) not seen
	A1	Correct unsimplified answer (with or without limits) Allow if area (2.4) not seen
	M1	Complete strategy for $\bar{y}$: use of moments equation with correct use of limits and division by area
	A1*	Use moments equation and given area to deduce given answer from correct working
(b)	M1	Use correct integral (with or without limits). Allow if area (2.4) not seen
	M1	Correct use of correct limits in an integral of the correct form and 2.4
	A1	Correct answer (1.0822….)
	M1	Complete strategy to find θ e.g find $\bar{x}$ and then use trig to find appropriate angle
	A1ft	Use the model to find a relevant angle. Follow their $\bar{x}$
	A1	2 s.f. or better 69.22...

Q10.

Question	Scheme	Marks	AOs
(a)	Correct strategy	M1	3.1a
	$8\bar{y}=\frac{1}{2}\int y^2\mathrm{d}x=\frac{1}{2}\int\left\{\frac{(x-2)^6}{16}+(x-2)^3+4\right\}\mathrm{d}x$	M1	2.1
	$=\frac{1}{2}\left[\frac{(x-2)^7}{7\times16}+\frac{(x-2)^4}{4}+4x\right]_0^4$	A1	1.1b
	$8\bar{y}=\frac{1}{2}\left[\frac{8}{7}+4+16+\frac{8}{7}-4-0\right]=\frac{64}{7}\ ,\quad \bar{y}=\frac{8}{7}$ *	A1*	2.2a
		(4)	
(b)	$8\bar{x}=\int\left(\frac{x(x-2)^3}{4}+2x\right)\mathrm{d}x$	M1	2.1
	$=\left[\frac{x(x-2)^4}{16}-\frac{(x-2)^5}{80}+x^2\right]_0^4$	A1	1.1b
	$=\frac{64}{16}-\frac{32}{80}+16-\frac{32}{80}=19.2$	M1	1.1b
	$\bar{x}=2.4$	A1	1.1b
	Complete strategy to find θ	M1	3.1a
	$\tan\theta=\frac{4-\bar{x}}{4-\frac{8}{7}}\left(=\frac{14}{25}\right)$	A1ft	3.4
	$\theta=29.2$ (Accept 29)	A1	1.1b
		(7)	
			(11 marks)

Question	Marks	Marking Guidance
(a)	M1	Complete strategy for $\bar{y}$: moments equation, use of limits and division by area
	M1	Moments equation to obtain terms of the correct form (with or without limits) The integral must be in terms of x only or y only Allow if area (8) not seen Might see $\int \frac{x^6}{16} - \frac{3x^5}{4} + \frac{15x^4}{4} - 9x^3 + 9x^2 \mathrm{d}x$ Or $\int xy\mathrm{d}y = \int 2y + y\left(4\left(y-2\right)\right)^{\frac{1}{3}}\mathrm{d}y$
	A1	Correct unsimplified answer (with or without limits) Allow if area (8) not seen $\left(\int xy\mathrm{d}y = 12.8\right)$
	A1*	Use moments equation and given area to deduce **given answer** from correct working
	(4)	
(b)	M1	Relevant integral in terms of x only or y only (with or without limits). Allow if area (8) not seen Could start with $\int xy\mathrm{d}x$ or $\int \frac{1}{2}x^2\mathrm{d}y$ Might see $\frac{x^4}{4} - \frac{6x^3}{4} + 3x^2 - 2x + 2x$
	A1	Correct unsimplified form after integration (with or without limits). Allow if area (8) not seen
	M1	Complete process to find $\bar{x}$ following relevant integral
	A1	Correct answer
	M1	Complete strategy to find θ e.g find $\bar{x}$ and then use trig to find appropriate angle
	A1ft	Use the model to find a relevant angle. Follow their $\bar{x}$
	A1	2 s.f. or better 29.2488...
	(7)	